Compressive Self-Noise Cancellation in Underwater Acoustics

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***Abstract*—The purpose of sonar is to detect the stealthy target in shallow water. The main barrier to locating the target is sonar's self-noise. Existing subspace-based noise suppression methods typically employ eigenanalysis-based methods involving high computational complexity. Recent approaches based on compressed sensing (CS) or sparse representations (SR) are computationally efficient. It is not straightforward to extend existing CS/SR-based methods for self-noise cancellation as, first, the energy of interference is much higher than the target, and second, it also exhibits similar sparsity properties. This work presents a novel method to combine the advantages of a subspace-based noise cancellation approach with low complexity of working with fewer CS measurements. Both target recovery and self-noise cancellation are done in the compressive domain only. Experimental results demonstrate the robustness of the proposed approach for both narrowband and broadband targets at very low signal-to-interference-noise (SINR).**

***Index Terms*—Self-noise cancellation, compressed sensing, underwater acoustics, sensor array**

I. Introduction

The problem of detecting an underwater target in the presence of background noise and estimating parameters such as range, depth, and bearing have been a point of research in the last few decades [1]–[4]. One of the major noise sources is the self-noise (interference) generated from the ship itself, which makes it challenging to perform passive signal processing onboard a moving ship to detect or locate a source. The standard approach to mitigate the effects of any interfering signal is to project the observed signal onto the subspace orthogonal to that of the interfering signal.

Existing methods proposed in various studies mainly differ in the computation of the noise subspace, which is estimated from eigenvectors of the correlation matrix of either the observed or interference signal [5]–[7]. If the correlation matrix is computed from observations, the number of sampled eigen directions corresponding to noisy subspace is often done empirically. To address this issue, work in [4] proposed a method based on eigenanalysis of cross-spectral density matrix (CSDM) of the data followed by beamforming each of the components. This helps identify the components with low target-to-interference power for robust detection of the target signal. When the energy of the interference signal is powerful compared to the target, the only reliable way of detection is to compute the noisy subspace from an estimate of the interference signal itself [7]. Nevertheless, any eigenanalysis-based approach suffers from high computational complexity, especially for high dimensional data from multiple sensors [8].

Further, in recent years, the sparse representation (SR) based methods have proved to be successful in a variety of underwater acoustic tasks such as direction-of-arrival (DOA) estimation and source localization [9]–[11]. These methods are based on the fact that it is relatively easy to find a sparse representation for target data given a suitable overcomplete basis (e.g., Fourier, wavelet) instead of noise (assumed to be additive). Other works further exploit the sparsity of signals to perform such tasks using very few random projections based on the principles of compressed sensing (CS) [12], [13]. For instance, work in [14], [15] proposed a compressive beamformer for DOA estimation. However, the CS/SR-based methods cannot simply be extended for self-noise cancellation (SNC). First, the interference energy is much higher than the target, and second, it exhibits similar sparsity properties. This work proposes a novel method to combine the advantages of a subspace-based noise cancellation approach with few CS measurements.The advantage of the proposed method is that it has reduced time and memory complexity compared to the high-dimensional subspace-based methods.

The rest of the paper is organized as follows: Section II describes array data model, The proposed compressive SNC framework follows this in Section III. Finally, the experimental results are detailed in Section IV, with a brief conclusion in Section V.

II. Array Data Model

Consider N sensor elements are arranged on a Uniform Linear Array (ULA) [1]. The received array signal $\mathbf{y}[n] \in \mathbb{R}^{T\times 1}$ at n^{th} sensor is a combination of the target signal, self-noise, and ambient noise as shown below :

$$\mathbf{y}[n] = \mathbf{A}(\theta)\mathbf{s}[n] + \mathbf{a}(\theta_0)s_o[n] + \mathbf{v}[n] \tag{1}$$

$\mathbf{A}(\theta) \in \mathbb{R}^{T\times J}$ is the steering matrix for signal vector $\mathbf{s}[n] \in \mathbb{R}^{J\times 1}$ at angle of incident θ of signal vector on array, s_0 is the self-noise (generated by the mother ship) associated with the steering vector $\mathbf{a}(\theta_0) \in \mathbb{R}^{T\times 1}$, here θ_0 is the self-noise bearing

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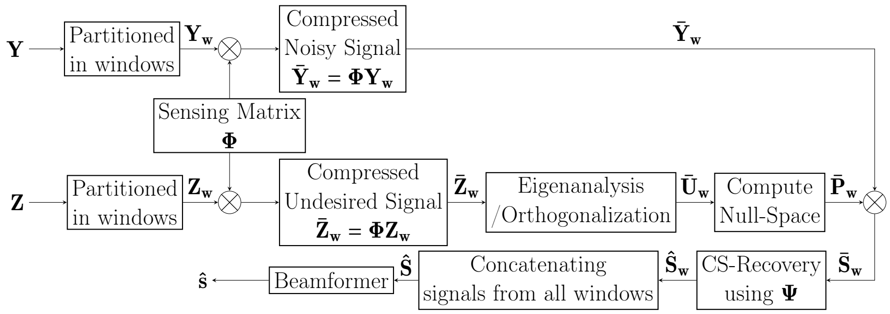


Fig. 1. Compressed self-noise cancellation via null-space projection

angle and $\mathbf{v}[n] \in \mathbb{R}^{T\times 1}$ represents the additive Gaussian noise. Received noisy signal at ULA is

$$\mathbf{Y} = [[\mathbf{y}(1)]\,,\,[\mathbf{y}(2)]\,,\,[\mathbf{y}(3)],\cdots[\mathbf{y}(N)]] \in [T\times N] \tag{2}$$

For simplicity, we denote the signal model in matrix form as:

$$\mathbf{Y} = \mathbf{S} + \mathbf{Z} \tag{3}$$

where the goal is to recover/detect the signal component $\mathbf{S}$ by removing undesired component $\mathbf{Z}$ due to the ambient and self-noise.

III. PROPOSED METHOD: COMPRESSIVE SNC

The conventional approach for self-noise cancellation maximizes signal-to-interference noise ratio (SINR) by utilizing the null space projection techniques. The optimal solution for detecting the desired signal signature by eliminating interference due to undesired signatures plus the noise is given as [6], [16]:

$$\hat{\mathbf{S}} = \mathbf{P}\mathbf{Y};\ \mathbf{P} = (\mathbf{I} - \mathbf{U}\mathbf{U}^{\dagger}) \tag{4}$$

where $\mathbf{P}$ is the projection matrix, and $\dagger$ denotes the pseudo-inverse. $\mathbf{U}$ are selected sampled orthogonal columns of $\mathbf{Z}$. The crucial difference between existing approaches lies in the computation of basis $\mathbf{U}$ which is estimated either from the correlation matrix of the interference or the observation matrix. In this work, we consider the former case (which is optimal here) as the energy of the interference/undesired signatures is powerful compared to the desired signature. From a numerical point of view, $\mathbf{U}$ is mostly estimated by applying orthogonal decomposition such as singular value decomposition (SVD) or rank-revealing QR decomposition and is chosen to be undercomplete as only the first few dominant directions (e.g., singular vectors) suffice to characterize the self-noise.

The inherently data-dependent nature of SVD/QR estimation involving expensive eigendecomposition [6] often hinders its use in severely resource-constrained settings such as underwater acoustics [17]. To address this issue, we propose to perform the null-space projection-based SNC in a compressed domain by projecting observed sensor data onto a random lower-dimensional subspace as highlighted in Fig.1. Here, we not only use compressed measurements to recover/detect the target signal but also perform the estimation of basis $\mathbf{U}$ for noise cancellation. To this aim, we reexpress matrices $\mathbf{Y}$ and $\mathbf{Z}$ as:

$$\begin{aligned}\mathbf{Y} &= [[\mathbf{y}(1)]\,,\,[\mathbf{y}(2)],\ldots[\mathbf{y}(T)]]^{\mathcal{T}},\\ \mathbf{Z} &= [[\mathbf{z}(1)]\,,\,[\mathbf{z}(2)],\ldots[\mathbf{z}(T)]]^{\mathcal{T}}\end{aligned} \tag{5}$$

where $\mathbf{y}(m)$ and $\mathbf{z}(m) \in \mathbb{R}^N$ are m^{th} row of $\mathbf{Y}$ and $\mathbf{Z}$ respectively. Here $[\ .\ ]^{\mathcal{T}}$ represents transpose of a matrix. Initially, We have partitioned $\mathbf{Y}$ and $\mathbf{Z}$ in 'B' number of windows.

$$\begin{aligned}\mathbf{Y_w}(m) &= [[\mathbf{y}((m-1)L+1)],\cdots,[\mathbf{y}((m-1)L+L)]]^{\mathcal{T}},\\ \mathbf{Z_w}(m) &= [[\mathbf{z}((m-1)L+1)],\cdots,[\mathbf{z}((m-1)L+L)]]^{\mathcal{T}}\end{aligned} \tag{6}$$

Subscript 'w' shows signal for one window. Here, $m = 1,2,...B$ and $T = BL$. L is the length of a window, where $\mathbf{Y_w} \in R^{L\times N}$ and $\mathbf{Z_w} \in R^{L\times N}$. In compressed sensing (CS), observations are measured using non-adaptive linear measurements:

$$\bar{\mathbf{Y}}_\mathbf{w} = \mathbf{\Phi}\mathbf{Y_w} = \mathbf{\Phi}(\mathbf{S_w} + \mathbf{Z_w}) = \mathbf{\Phi}(\mathbf{\Psi}\mathbf{A_w} + \mathbf{Z_w}) \tag{7}$$

$$\bar{\mathbf{Z}}_\mathbf{w} = \mathbf{\Phi}\mathbf{Z_w} \tag{8}$$

where $\mathbf{\Phi} \in \mathbb{R}^{l\times L}(l << L)$ denotes the sensing matrix consisting of 'l' random orthonormal random vectors. We assume signal from each sensor exhibit a k-sparse representation (as columns of $\mathbf{A_w}$) in a basis $\mathbf{\Psi}$ [12]. Study in [18] showed that under the mild assumption of the eccentricity of dominant eigenvalues, the eigenvectors of covariance matrix $\mathbf{Z_w^T}\mathbf{Z_w}/L$ in original domain and $\mathbf{\Phi}(\mathbf{Z_w^T}\mathbf{Z_w})\mathbf{\Phi^T}/l$ are related. By exploiting this property, the SNC procedure in (4) can be performed in low-dimensional compressive space as:

$$\bar{\mathbf{S}}_\mathbf{w} = \bar{\mathbf{P}}_\mathbf{w}\mathbf{\Phi}\mathbf{Y_w};\ \bar{\mathbf{P}}_\mathbf{w} = (\mathbf{I} - \bar{\mathbf{U}}_\mathbf{w}\bar{\mathbf{U}}_\mathbf{w}^{\dagger}) \tag{9}$$

where $\mathbf{I}$ is the identity matrix. $\bar{\mathbf{P}}_\mathbf{w}$ and $\bar{\mathbf{U}}_\mathbf{w}$ are the projection and the orthogonal matrices respectively in the compressive domain. The processed measurements $\bar{\mathbf{S}}_\mathbf{w}$ can be considered as an approximation of CS measurements of signal component $\mathbf{S}_w$. It has been shown that stable recovery of $\mathbf{S}_w$ in terms of its sparse representation $\mathbf{A}_w$ is possible if $\mathbf{\Phi}$ satisfies the restricted isometry property (RIP) and is incoherent with basis

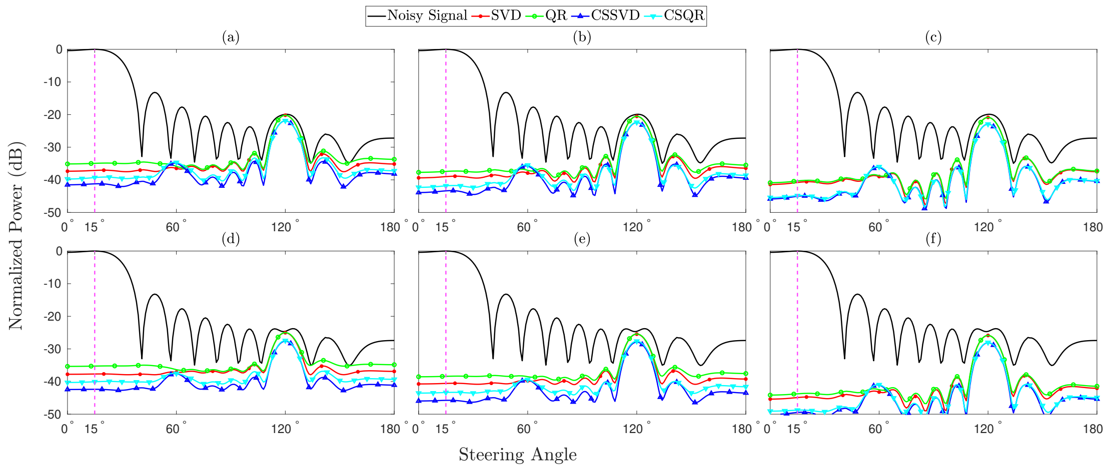


Fig. 2. Beampattern for noisy and recovered NB stationary signal. Target bearing is at 120°, and self-noise bearing is 15°. (a,b,c) at SINR -20dB and (d,e,f) at SINR -25dB using (a,d) top 10, (b,e) top 20 and (c,f) top 30 sampled orthogonal vectors (SOV), respectively. The compression ration $l/L = .2$ is used in case of CSSVD and CSQR methods.

$\mathbf{\Psi}$ [12], [13]. The estimation of the signal matrix requires solving N independent inverse-problems of the form:

$$\begin{aligned} &\text{argmin}\|\bar{\mathbf{S}}_{\mathbf{w}} - \mathbf{\Phi}\mathbf{\Psi}\mathbf{A}_{\mathbf{w}}\|_F^2 \ \text{ s.t. } \ \|\mathbf{a}_i\|_0 \leq k, \\ &\mathbf{A}_w = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \ldots]; \quad \hat{\mathbf{S}}_{\mathbf{w}} = \mathbf{\Psi}\hat{\mathbf{A}}_{\mathbf{w}} \end{aligned} \tag{10}$$

where $\|.\|_0$ denotes the ℓ_0-norm, $\mathbf{a}_i$ are the columns of $\mathbf{A}_w$, and k denotes the cardinality of a vector. (10) is a non-convex problem [19]–[21] and its solution can be obtained by matching pursuit-based greedy algorithms or by relaxing the sparsity constraints and using ℓ_1-norm based solvers instead [12]. In this work, we employ discrete-time cosine transform (DCT) as the sparsifying basis and random-ortho Gaussian matrix as measurement matrix as it satisfies incoherence or RIP conditions with high probability [13]. We denote the two-step procedure in (9) and (10) as compressive self-noise cancellation method. Finally, the complete recovered signal can be obtained by concatenating recovered signal for all windows:

$$\hat{\mathbf{S}} = [\hat{\mathbf{S}}_{\mathbf{w}}(1), \hat{\mathbf{S}}_{\mathbf{w}}(2), \ \cdots \hat{\mathbf{S}}_{\mathbf{w}}(n)]^{\mathcal{T}} \tag{11}$$

This is followed by post-processing using a delay-and-sum beamformer [8] to get beamformed output $\hat{\mathbf{s}}$. Thus, the proposed method reduces time complexity to $\mathcal{O}(L^2N)$ as compared to the high-dimensional subspace-based method ($\mathcal{O}(l^2N)$).

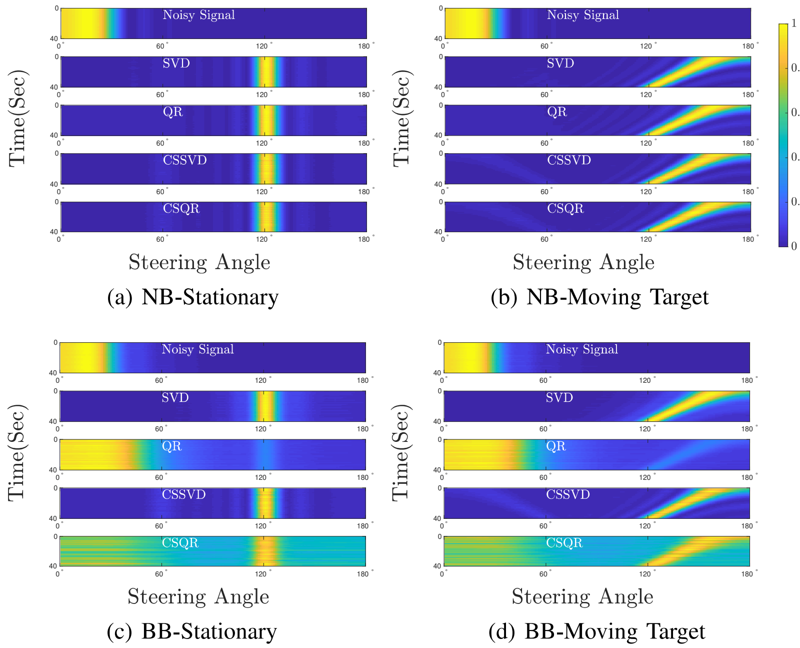
IV. Experimental Results

A. Experimental Setup

An experimental study is performed to evaluate the performance of the proposed compressive SNC approach for target detection in underwater acoustics. The simulation is done for Narrowband (NB) and Broadband (BB) Signals, both with stationary and moving targets in the presence of Gaussian ambient noise. Target and self-noise bearing are set to be 120° and 15°, respectively, with moving target bearing varying at 1° per second. The ULA contains 32 sensors that capture the signal at a sampling rate of 12800 Hz over an observation time of the 40s. For signal recovery, we measure and process the signal using non-overlapping rectangular windows of size 80ms. The projection matrix $\bar{\mathbf{P}}$ is estimated from CS measurements using SVD and QR decomposition, where we only sample a few top consecutive orthogonal vectors to form basis $\bar{\mathbf{U}}$. To recover the signal from projected CS samples $\bar{\mathbf{Y}}$ we employ greedy sparse recovery algorithms. In particular, we experimented with compressive sampling matching pursuit (CoSaMP) [22], and orthogonal matching pursuit (OMP) [23] algorithms and found OMP to be more robust in recovery both at low SNR and less number of measurements. The recovered signal from all sensors is beamformed using a delay-and-sum beamformer, and the recovery performance is reported using: 1) plot of normalized beam power as a function of steering angle; and 2) waterfall display (WD) of detected power signature as a function of time and steering angle. We denote the plots/curves corresponding to recovered signal after noise-cancellation in the original domain as SVD or QR and in the compressed domain as CSSVD or CSQR.

B. Results for Narrowband Signal

In this experiment, we consider the case where both target and self-noise are narrowband with a single signal frequency component of 1300 Hz and interference frequency of 1200 Hz. Fig. 2 shows the beampattern corresponding to the recovered target and the observed noisy signal at SINR of -20dB and -25dB using top 10, 20 and 30 sampled orthogonal vectors (SOV). Our baseline here is the SVD method, which can be observed to have good noise-cancellation and target localization with most of the power concentrated in the main lobe centered at 120° (see Fig. 2(a)). The QR method also exhibits comparable target recovery and localization. In contrast, both CSSVD and CSQR methods have good target localization and better noise-cancellation performance in terms of lower power



(a) NB-Stationary (b) NB-Moving Target

(c) BB-Stationary (d) BB-Moving Target

Fig. 3. Waterfall display for noisy and recovered [NB/BB] stationary and moving target at SINR -20 dB using top 20 sampled orthogonal vectors.

in side lobes. Note that the main lobe power in the case of CSSVD/CSQR is slightly less than SVD/QR methods and is a function of the compression ratio l/L (see Section IV-D for more details). We have also analyzed the impact of a number of SOV of $\bar{\mathbf{U}}$ to form a projection matrix $\bar{\mathbf{P}}$. While the target localization is comparable, one can observe that as we sample more vectors, the uniformly distributed side lobe power becomes more concentrated at certain steering angles. These results are consistent for different SINR and both stationary and moving targets. We further evaluate the Self Noise - Power Level Reduction (SN-PLR) and Target Power Loss (TPL). It is the absolute difference in power output (in dB) between the recovered and noisy signals at self-noise and target bearing, respectively. The SN-PLR (TPL) scores at SINR-20 dB and -25 dB are illustrated in Table I and II respectively at $l/L = .2$. Although the CSSVD/CSQR show higher TPL, the target is still being localized with respectable suppression in self-noise (higher SN-PLR compared to SVD/QR). To demonstrate the bearing history of the recovered target at a specific time, we show the WD plots in Fig. 3(a) and (b). In particular, we only show WD plots for moving targets to visualize the temporal behavior better. Observe maximum power at $120°$ for stationary target and how the signature is localized from $120°$ to $180°$ throughout the 40s in WD plots. We see that CSSVD and CSQR methods can recover, localize, and track the target while simultaneously suppressing the ambient and self-noise even for moving targets.

C. Results for Broadband Signal

This experiment considers a more complex broadband case with the frequency range for both target and interference being 100Hz-2000Hz. Due to space constraints, we only report the results using the WD plot in Fig. 3(c) and (d). For stationary targets, it can be observed that both SVD and CSSVD methods have comparable performance in terms of noise cancellation and target localization. However, the QR method is unable to recover the target, which demonstrates that the choice of the orthogonal subspace is crucial. Here the sample vectors do not seem to correspond to self-noise leading to undesirable results. Interestingly, the CSQR method can still locate the target, and although self-noise is not fully canceled, it is distributed along with other bearing angles. We observe similar trends for the case of moving target where the SVD method performs the best, followed by CSSVD, CSQR, and QR methods, respectively. These results demonstrate the advantage of exploiting the sparsity to achieve SNC in the compressive domain.

TABLE I
SN-PLR (TPL) FOR NB STATIONARY TARGET AT SINR -20 DB

SOV	**SVD**	**QR**	**CSSVD**	**CSQR**
10	37.12 (0.28)	34.95 (**0.27**)	**41.23** (1.99)	39.31 (1.95)
20	39.02 (0.61)	37.42 (**0.59**)	**43.38** (2.50)	41.85 (2.44)
30	40.81 (**0.93**)	40.35 (**0.93**)	**45.06** (2.99)	44.86 (2.99)

TABLE II
SN-PLR (TPL) FOR NB STATIONARY TARGET AT SINR -25 DB

SOV	**SVD**	**QR**	**CSSVD**	**CSQR**
10	37.69 (5.10)	35.26 (**5.05**)	**42.38** (7.68)	40.07 (7.52)
20	40.58 (5.53)	38.40 (**5.48**)	**45.76** (7.69)	43.35 (7.71)
30	44.91 (5.94)	43.77 (**5.92**)	**49.49** (8.16)	48.77 (8.09)

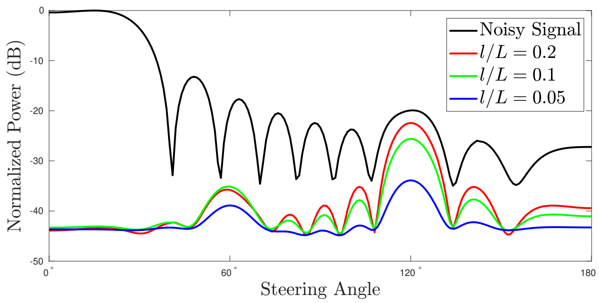


Fig. 4. Beampattern of the noisy & recovered signal at SINR -20dB using top 20 orthogonal vectors as a function of compression ratio.

D. Impact of compression ratio

In this experiment, we assess the recovery performance of the proposed compressive SNC method as a function of compression ratio (l/L). This is important to understand the trade-off between computational complexity using fewer CS samples vs. the target recovery/localization performance. In particular, we consider the case of NB stationary target where the SNC is performed using the CSSVD method at compression ratio of 0.2, 0.1, 0.05, respectively. Fig. 4 shows the beampattern corresponding to the observed noisy signal at SINR of -20dB and the recovered target using top 20 orthogonal vectors. It can be inferred that even with very few measurements, especially $l/L = .05$, the proposed method is able to localize the target. However, the ability to resolve the main lobe at $120°$ from the other side lobe improves as the ratio l/L increases.

V. Conclusion

We have presented a CS-based approach for self-noise cancellation and target localization in this work. Consistent with existing studies, we demonstrate the efficacy of the CS approach in exploiting the sparsity of the target for a robust recovery in the case of both narrowband and broadband signals. The novelty of our approach lies in the combination of the subspace-based noise-cancellation approach with CS-based target localization in the presence of self and ambient noise. Self-noise typically has much higher power than target and also exhibits sparse properties. Hence, we first employ null-space projection in the compressive domain to suppress noise followed by conventional CS-based target recovery. Finally, we experimentally demonstrated that working with various orthogonal decomposition methods to estimate noisy subspace in the compressed domain is more robust than working directly in the original high-dimensional signal domain. Our future work will focus on optimizing the sensing matrix for multiple target localization.

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